Next to leading order evolution of SIDIS processes in the forward region*

A. Daleo[1,†] and R. Sassot[2]

[1]*Laboratorio de Física Teórica*
Departamento de Física, Facultad de Ciencias Exactas
Universidad Nacional de La Plata
C.C. 67 - 1900 La Plata, Argentina
[2]*Departamento de Física, Universidad de Buenos Aires*
Ciudad Universitaria, Pab.1 (1428) Buenos Aires, Argentina

We compute the order α_s^2 quark initiated corrections to semi-inclusive deep inelastic scattering extending the approach developed recently for the gluon contributions. With these corrections we complete the order α_s^2 QCD description of these processes, verifying explicitly the factorization of collinear singularities. We also obtain the corresponding NLO evolution kernels, relevant for the scale dependence of fracture functions. We compare the non-homogeneous evolution effects driven by these kernels with those obtained at leading order accuracy and discuss their phenomenological implications.

PACS numbers: 12.38.Bx, 13.85.Ni
Keywords: Semi-Inclusive DIS; perturbative QCD; Fracture functions

Introduction

Fracture functions play a fundamental role in the description of one particle inclusive deep inelastic scattering [1, 2, 3, 4]. They have been incorporated into the analysis of a variety of specific processes, although only up to leading order accuracy [5, 6, 7, 8, 9], since their scale dependence, which includes non homogeneous effects, had not been studied at higher orders until recently. At the lowest order in QCD perturbation (LO), these functions account for processes in which the final state particle is produced in the direction of the target remnant, and can be interpreted as the probability to find a parton in an already fragmented nucleon target [1]. Their non homogeneous scale dependence is found to be strongly suppressed at LO in most of the kinematic domain, particularly in the kinematic region covered by the present experiments, and thus neglected. However, there are no indications if this suppression is also present at NLO level.

In reference [10] the gluon initiated corrections to one particle inclusive deep inelastic scattering were computed up to order α_s^2 for the first time. There it was shown that at variance with the fully inclusive case, in the one particle inclusive cross section at this order, collinear configurations lead to singular distributions in more than one variable which can not be handled with the standard procedure. However a suitable method for the prescription of the characteristic entangled singularities was developed, verifying explicitly the factorization of collinear singularities and obtaining the corresponding non homogeneous evolution kernels. Regarding the phenomenological consequences of these corrections, it was found that the non homogeneous contributions to the evolution equations at NLO may be noticeably larger than the ones found at LO. Of course, gluon initiated corrections do not provide all the kernels relevant for the scale dependence of fracture functions, a consistent assessment of this dependence requires also the consideration of quark initiated processes at the same order.

In the present paper we extend the computation of the $\mathcal{O}(\alpha_s^2)$ corrections to one particle inclusive DIS for the case of quarks in the initial state. These last processes involve two caveats, a more complex singularity structure in the partonic cross sections due to the presence of virtual corrections, and complications associated to the flavor structure and decomposition for these corrections.

Quark initiated corrections imply the computation of several real and virtual contributions which lead to a much more complicated singularity structure than in the gluon initiated case. The entanglement of singularities is precisely the most difficult point in the computation of higher order corrections to one particle inclusive processes. Nevertheless, we show that they can be handled extending the strategy implemented for the latter case.

Regarding the flavor structure, the explicit factorization of collinear singularities in the one particle inclusive cross section does not allow to obtain all the evolution kernels individually, only certain combinations of them can be

*Partially supported by CONICET, Fundación Antorchas, UBACYT and ANPCyT, Argentina.
†Electronic address: daleo@fisica.unlp.edu.ar

extracted. Consequently, at this point it is not possible to achieve a complete flavor discrimination in the evolution of fracture functions. However it is possible to define certain singlet and non singlet flavor combinations, identify their respective evolution kernels, and obtain the evolution of the cross section.

As in the case of gluon initiated corrections, NLO contributions to the non homogeneous evolution initiated by quarks are found to be comparable or even larger than the LO ones in some kinematical regions. In order to assess the significance of non homogeneous contributions relative to homogeneous ones in the scale dependence, we implement the full evolution equations for $M_{q,\pi^+/P}$ using as input a model estimate for these fracture functions at an initial scale. The results we found indicate large non homogeneous effects at small x_L and x_B but also suggest suppression for both LO and NLO contributions otherwise. On the other side, this estimate does not take into account the NLO non homogeneous kernels for gluon fracture functions, which show up at order α_s^3 in the cross section. These kernels contribute indirectly to M_q through the homogeneous terms in the evolution but have not been computed yet.

In the following section we summarize the relevant kinematics and details about the phase space integration for the $\mathrm{O}(\alpha_s^2)$ contributions to the cross section, together with the conventions and notation adopted. In section II we compute the corresponding real and virtual amplitudes associated to each of the quark initiated processes, we discuss the nature of the singularities that contribute to them and the prescriptions required. Then, in section III we analyse the factorization of collinear singularities, and issues related to the flavor decomposition. There, we also present the evolution kernels obtained and derive the evolution equations. Section IV discuss the phenomenological implications of the new corrections and finally we end with our conclusions.

I. NOTATION AND KINEMATICS

We begin by reviewing the main features of the QCD description of one particle inclusive deep inelastic scattering processes. We consider the process

$$l(l) + P(P) \longrightarrow l'(l') + h(P_h) + X, \tag{1}$$

where a lepton of momentum l scatters off a nucleon of momentum P with a lepton of momentum l' and a hadron h of momentum P_h tagged in the final state. The cross section for this process can be written as [2]

$$\begin{aligned} \frac{d\sigma}{dx_B\, dy\, dv_h\, dw_h} = & \sum_{i,j=q,\bar{q},g} \int_{x_B}^1 \frac{du}{u} \int_{v_h}^1 \frac{dv_j}{v_j} \int_0^1 dw_j\, f_{i/P}\left(\frac{x_B}{u}\right) D_{h/j}\left(\frac{v_h}{v_j}\right) \frac{d\hat{\sigma}_{ij}}{dx_B\, dy\, dv_j\, dw_j}\, \delta(w_h - w_j) \\ & + \sum_i \int_{\frac{x_B}{1-(1-x_B)v_h}}^1 \frac{du}{u}\, M_{i,h/P}\left(\frac{x_B}{u}, (1-x_B)v_h\right) (1-x_B) \frac{d\hat{\sigma}_i}{dx_B\, dy}\, \delta(1-w_h)\,, \end{aligned} \tag{2}$$

where the sum is over all parton species. The leptonic final state, in the one photon exchange approximation, is described by the usual deep inelastic variables

$$Q^2 = -q^2 = -(l'-l)^2\,, \quad x_B = \frac{Q^2}{2P\cdot q}\,, \quad y = \frac{P\cdot q}{P\cdot l}\,, \quad S_H = (P+l)^2\,, \tag{3}$$

whereas, to describe the hadronic final state, we define

$$v_h = \frac{E_h}{E_0(1-x_B)}\,, \qquad w_h = \frac{1-\cos\theta_h}{2}\,. \tag{4}$$

E_h and E_0 in the preceding formulae are the energies of the final state hadron and of the incoming proton in the $\vec{P} + \vec{q} = 0$ frame, respectively. θ_h is the angle between the momenta of the hadron and that of the virtual photon in the same frame. The convolution variables v_i and w_i in equation (2) are the partonic analogs of v_h and w_h and u is related to the fraction of momentum of the incoming parton ξ by $\xi = x_B/u$.

The two terms in the r.h.s. of equation (2) stand for 'current' and 'target' fragmentation processes respectively. In the first one $d\hat{\sigma}_{ij}$ represents the partonic cross section for the process $l + i \to l' + j + X$, whereas $f_{i/P}$ and $D_{h/j}$ are the usual partonic densities and fragmentation functions. In the second term the cross sections for the inclusive process $l + i \to l' + X$, $d\hat{\sigma}_i$, are convoluted with the fracture functions $M_{i,h/P}$ which give the probability of finding parton i in an already fragmented target. In this last case, the hadron is produced in the direction of the incoming proton (notice the $\delta(1-w_h)$ in the last factor of the second term).

The partonic cross sections $d\hat{\sigma}_i$ and $d\hat{\sigma}_{ij}$ can be computed order by order in perturbation theory from the corresponding parton-photon squared matrix elements, $i + \gamma \to X$ and $i + \gamma \to j + X$ respectively. The totally inclusive

cross sections up to order α_s^2, are well known and can be found in reference [11]. The lowest order one-particle inclusive cross section (zeroth-order in α_s) is given by

$$\frac{d\hat{\sigma}^{(0)}_{qq,M}}{dx_B\, dy\, dv\, dw} = \frac{d\hat{\sigma}^{(0)}_{\bar{q}\bar{q},M}}{dx_B\, dy\, dv\, dw} = \frac{4\pi\,\alpha^2\, e_q^2}{2\, x_B\, S_H}\,(2+\epsilon)\, Y_M\, \delta(1-u)\delta(1-v)\delta(w)\,, \tag{5}$$

where the outgoing quark is always produced backwards ($w = 0$). The index M stands for metric, longitudinal contributions are absent at tree level, and $Y_M = (1 + (1-y)^2)/2y^2$. The result correspond to $d = 4 + \epsilon$ space-time dimensions.

The order-α_s corrections to the one-particle inclusive cross sections are also known. Expressions for the singular and finite terms computed within the framework of dimensional regularization[12] can be found in [2] for the unpolarized scattering, and in [4] for the polarized case. Finite contributions up to order ϵ, needed for the factorization of collinear singularities at $\mathcal{O}(\alpha_s^2)$, are given in [10].

At order-α_s the fragmenting parton can be produced in any direction, including the forward one ($w = 1$), but all singular contributions come either from the backward or from the forward direction. The former terms are factorized into partonic densities and fragmentation functions, whereas the forward singularities can only be factorized in the redefinition of fracture functions. This factorization gives rise, as we have already mentioned, to non-homogeneous terms in the evolution equations of fracture functions [1].

At order-α_s^2 the partonic cross sections develop singularities in every direction in space. They receive contributions from both virtual and real amplitudes whose matrix elements have to be integrated over the phase space of the unobserved partons (i.e. those partons that are not attached to the fragmentation function). At this order, real contributions have three particles in the final state, and the corresponding phase space can be written as:

$$\begin{aligned} dPS^{(3)} \;=\; & \frac{Q^2}{(4\pi)^4}\,\frac{1}{\Gamma(1+\epsilon)}\left(\frac{Q^2}{4\pi}\right)^{\epsilon}\left(\frac{1-x_B}{x_B}\right)^{1+\epsilon/2}\left(\frac{u-x_B}{x_B}\right)^{\epsilon/2} v^{1+3\,\epsilon/2}\theta(w_r-w)\,(w_r-w)^{\epsilon/2}\,w^{\epsilon/2} \\ & \times(1-w)^{\epsilon/2} dv\, dw\, \sin^{1+\epsilon}\beta_1\, \sin^{\epsilon}\beta_2\, d\beta_1\, d\beta_2\,. \end{aligned} \tag{6}$$

The angles β_1 and β_2 are the polar and azimuthal angle of the pair of unobserved partons in their center of mass frame. In order to enforce the kinematical constraints and deal with the singularities in the prescription stage, it is convenient to break up the integration range into three regions, $R = \mathrm{B0} \cup \mathrm{B1} \cup \mathrm{B2}$ with

$$\begin{aligned} &\mathrm{B0} = \{u \in [x_B, x_u],\; v \in [v_h, a],\; w \in [0,1]\} \\ &\mathrm{B1} = \{u \in [x_B, x_u],\; v \in [a, 1],\; w \in [0, w_r]\} \\ &\mathrm{B2} = \{u \in [x_u, 1],\; v \in [v_h, 1],\; w \in [0, w_r]\}\,, \end{aligned} \tag{7}$$

where

$$x_u = \frac{x_B}{(x_B + (1-x_B)v_h)}\,,\quad a = \frac{(1-u)\,x_B}{u\,(1-x_B)}\,,\quad w_r = \frac{(1-v)\,(1-u)\,x_B}{v\,(u-x_B)}\,. \tag{8}$$

B1 and B2 receive contributions from both $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2)$ processes, while B0 has only $\mathcal{O}(\alpha_s^2)$. Virtual contributions can have either one or two particles in the final state. As it will be discussed in the next section, only diagrams with two particles in the final state give rise to contributions in the forward direction. In this case, the phase space can be conveniently cast into

$$dPS^{(2)} \;=\; \frac{1}{8\pi\,\Gamma(1+\epsilon/2)}\left(\frac{Q^2}{4\pi}\right)^{\epsilon/2}\frac{u\,(1-x_B)}{u-x_B}\left(\frac{1-x_B}{x_B}\right)^{\epsilon/2} v^{\epsilon} w^{\epsilon/2}(1-w)^{\epsilon/2}\,\delta(w_r-w)\,dv\,dw\,, \tag{9}$$

where now the integration is restricted to the region $V = \mathrm{B1} \cup \mathrm{B2}$. Notice that, for these contributions, the energy and angle variables, v and w, are correlated by the δ function which constraints the variables to the surface $w = w_r$. This distinctive feature of the two particle phase space already appeared at $\mathcal{O}(\alpha_s)$ [2].

II. ORDER α_s^2 PARTONIC CROSS SECTIONS

Order-α_s^2 contributions to the one-particle inclusive cross sections come from the following partonic reactions:

$$
\begin{array}{ll}
\text{Real contributions} & \left\{ \begin{array}{lll} \gamma + q(\bar{q}) & \rightarrow & g + g + q(\bar{q}) \\ \gamma + q_i(\bar{q}_i) & \rightarrow & q_i(\bar{q}_i) + q_j + \bar{q}_j \quad (i \neq j) \\ \gamma + q_i(\bar{q}_i) & \rightarrow & q_i(\bar{q}_i) + q_i + \bar{q}_i \\ \gamma + g & \rightarrow & g + q + \bar{q} \end{array} \right. \\
\text{Virtual contributions} & \left\{ \begin{array}{lll} \gamma + q(\bar{q}) & \rightarrow & q(\bar{q}) \\ \gamma + q(\bar{q}) & \rightarrow & g + q(\bar{q}) \\ \gamma + g & \rightarrow & q + \bar{q} \end{array} \right.
\end{array}
\tag{10}
$$

where any of the outgoing partons can fragment into the final state hadron h. Gluon initiated reactions were already discussed in deep in reference [10]. In this section we analyze quark initiated processes, and examine the the nature of the singularities that they involve. These contributions are computed in $d = 4 + \epsilon$ dimensions, in the Feynman gauge, and considering all the quarks as massless as in reference [10]. Algebraic manipulations were performed with the aid of the program MATHEMATICA [13] and the package TRACER [14] to perform the traces over the Dirac indices.

Taking into account all the alternatives for one parton undergoing fragmentation into the final state hadron h, the reactions in equation (10) lead to the following partonic cross sections: σ_{qg}, $\sigma_{q_iq_i}$, $\sigma_{q_i\bar{q}_i}$, $\sigma_{q_iq_j}$, $\sigma_{q_i\bar{q}_j}$, and their charge conjugates, exhausting all the possible combinations. The first index labels the parton that initiates the reaction, while the second stands for the one that precedes the final state hadron. Notice that for the last four cross sections, with quarks in both the initial and the hadronized state, the are several flavor combinations, each leading to characteristic singular contributions.

Contributions to the σ_{qg} cross sections come from the first and the sixth reactions in equation (10), corresponding to real and virtual diagrams respectively. These are shown in figures 1 and 2. Virtual contributions are obtained from the interference of diagrams in figure 2 with those of single bremsstrahlung at order α_s. The diagrams in the second row of Figure 1 need to be taken into account due to the symmetric character of the outgoing gluons wave function which also contributes with a factor 1/2 to the squared amplitude.

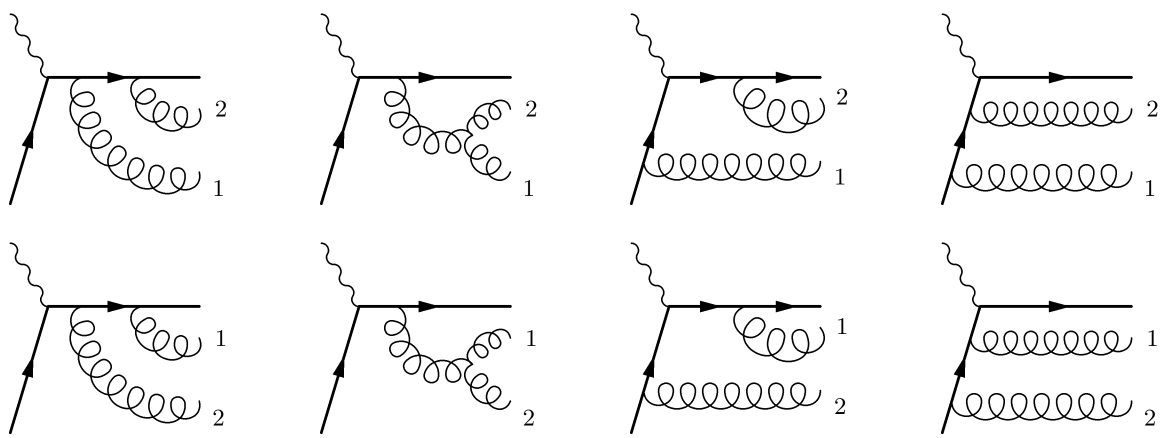

FIG. 1: Real contributions at order α_s^2 for the $\gamma + q(\bar{q}) \rightarrow g + g + q(\bar{q})$ process

For the remaining contributions, where a final state quark hadronizes, it is necessary to discriminate whereas this last quark carries the same flavor of the initial state quark or not. Contributions to the $\sigma_{q_iq_j}$ and $\sigma_{q_i\bar{q}_j}$ cross sections with $i \neq j$, arise form the diagrams in the first row of Figure 3 corresponding to the second reaction in equation (10). The amplitudes of the first two diagrams in this row are proportional to e_i, whereas the two last amplitudes are proportional to e_j. Consequently, the above mentioned cross sections can be decomposed into three pieces, proportional to e_i^2, e_j^2 and $e_i\, e_j$ respectively. Since these cross sections first appear at order-α_s^2 they receive no virtual contributions at this order. As the amplitudes are symmetric under the interchange of the quark-antiquark pair of flavor j, the relation $\sigma_{q_iq_j} = \sigma_{q_i\bar{q}_j}$ is satisfied.

Finally we have the contributions to $\sigma_{q_iq_i}$ and $\sigma_{q_i\bar{q}_i}$, where the initial and hadronizing quarks carry the same flavor. In this case, the relevant reactions are the first three, the fifth and the sixth in equation (10) corresponding to diagrams in Figures 1 and 3 (contributions in both $i \neq j$ and $i = j$ cases) for the real contributions and diagrams in Figures 2 and 4 for the one and two loop virtual corrections respectively. Notice, that in spite of having the same label for

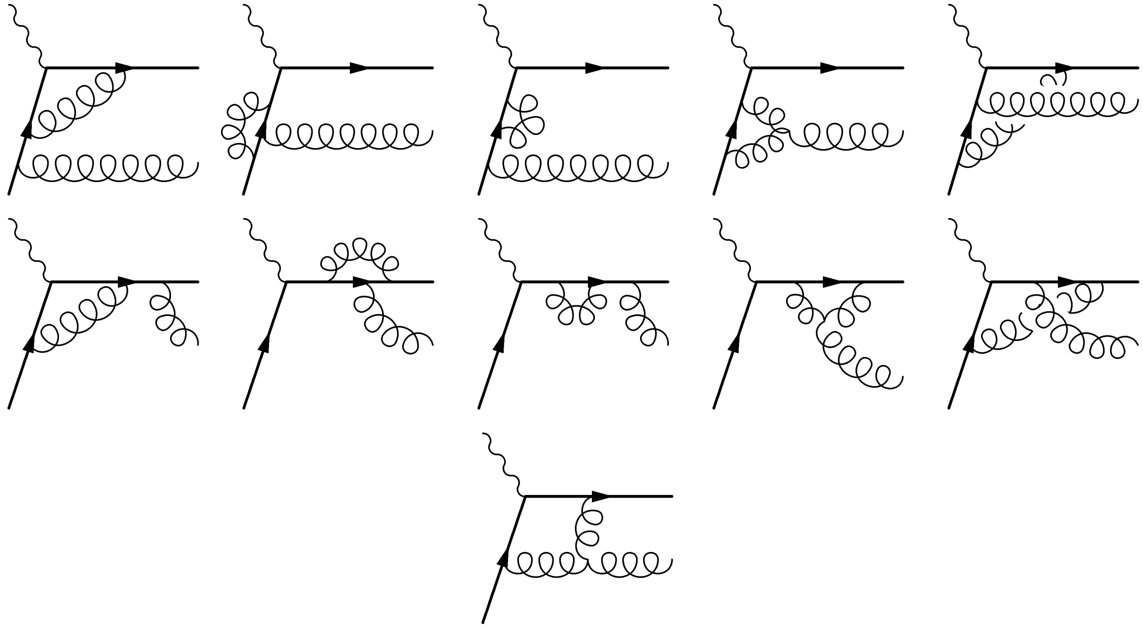

FIG. 2: Virtual contributions at order α_s^2 for the $\gamma + q(\bar{q}) \to g + q(\bar{q})$ process

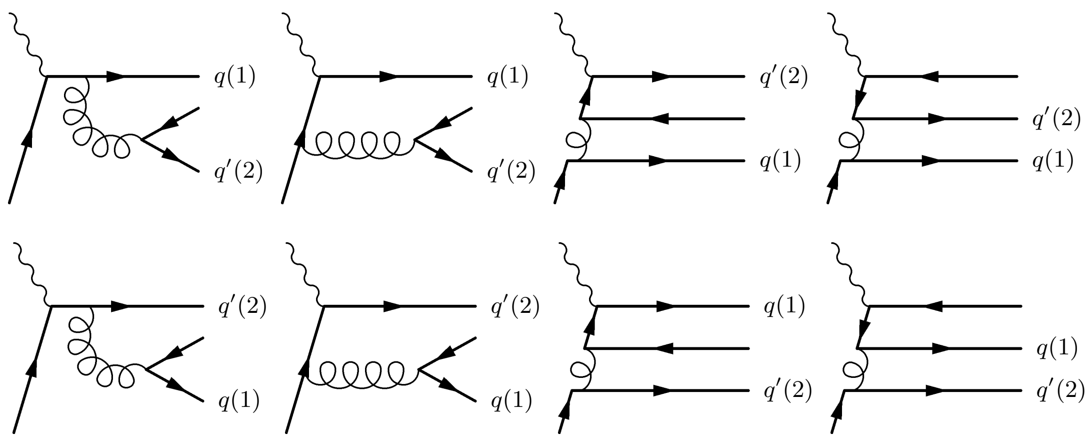

FIG. 3: Real contributions at order α_s^2 for the $\gamma + q(\bar{q}) \to q' + \bar{q}' + q(\bar{q})$ process. Diagrams in the second row only contribute in the case $q = q'$

initial and hadronizing quarks, these cross sections include also terms proportional to e_i^2, $\sum_{j\neq i} e_j^2$ and $\sum_{j\neq i} e_i\, e_j$, since the quark which enters the electromagnetic vertex in the third and fourth diagrams in figure 3 can be of any flavor. Regarding the process $\gamma + q_i \to q_i + q_i + \bar{q}_i$, diagrams in the second row in Figure 3 have a relative minus sign respect to those in the first row due to the antisymmetry of the wave function of the identical outgoing quarks which also contributes with a factor $1/2$ to the matrix element.

The angular integration of the matrix elements for the real contributions can be performed with the standard techniques [15], taking into account the additional complications of the one particle inclusive case: the necessity of collecting to all orders the potentially singular factors in the three particle final state integrals, as explained in [10]. For the integrals that are known to all orders in ϵ, this is not a problem, while for those which are only known up to a given order a careful treatment is required. In Appendix A we include a list with the integrals that appeared in the present calculation.

Once the angular integrals are performed, matrix elements are still distributions in u, v and w, regulated by the parameter ϵ. In order to write these contributions, specifically their potentially singular pieces, in a consistent way, they need to 'prescribed', i.e. written as an expansion in powers of ϵ with integrable coefficients. At variance with the totally inclusive case, where the integration over final states leads only to singularities in the u variable, in this

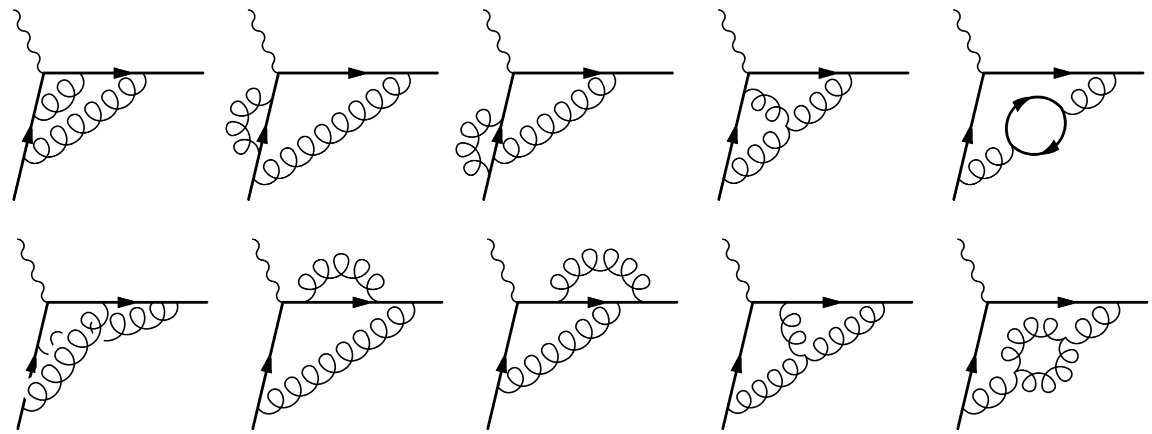

FIG. 4: Two loops contributions at order α_s^2 for the $\gamma + q(\bar{q}) \to q(\bar{q})$ process (quark form factor).

case they appear along various curves in the residual phase space. Contributions with singularities along one or two intersecting curves can be managed with the technique developed in [10]. However, processes with identical quarks in both initial and hadronizing states lead to singularities along three intersecting curves, requiring a different prescription strategy. This distinctive feature of the $\sigma_{q_i q_i}$ cross section is related to the fact that it give rise to quartic poles in ϵ, which cancel out between real and virtual contributions.

As an example, we consider the subprocess $\gamma + q \to q + g + g$, where the final state quark fragments into hadron h. The quark propagators coming from the interference of the first and fourth diagrams in the first row of Figure 1 contribute with a factor

$$\frac{1}{(k_1+p_f)^2\,(p_i-k_2)^2\,(k_1+k_2+p_f)^2\,(p_i-k_1-k_2)^2}\,, \tag{11}$$

where p_i and p_f are the momenta of the incoming and outgoing quarks respectively, while k_1 and k_2 are those of the gluons. In the center of mass frame of the two gluons, the integration over their phase space involves only the first two denominators. In this frame, the two other denominators do not depend on the angles β_1 and β_2, defined in equation (6). In this particular case, the angular integrals can be performed to all orders in ϵ and have single poles in $\epsilon = 0$. The most singular terms can be written as

$$\frac{1}{\epsilon}\, w^{-1+\epsilon}\,(w-w_0)^{-1+\epsilon/2}\,(w_r-w)^{\epsilon/2}\,\frac{f(u,v,w,\epsilon)}{1-u}\,, \tag{12}$$

where $f(u,v,w)$ is a regular function in all the integration region, specifically finite when $\epsilon \to 0$ and $w_0 = -(1-v)/v$. Notice the presence of singularities along $w=0$, $w=w_0$ and $u=1$, and that these three curves intersect at the point $u=v=1$, $w=0$.

In region B0 only the factor $w^{-1+\epsilon}$ in equation (12) is potentially divergent, since $v \le a < 1$ and $u \le x_u < 1$. This allows the use of the standard prescription recipe, which lead to a double pole accompanied by a $\delta(w)$. In region B1, the singularity at $w = w_0$ need also to be taken into account, however this can be managed with the approach described in [10]. In this case we obtain

$$\begin{aligned} w^{-1+\epsilon}\,(w-w_0)^{-1+\epsilon/2}\,(w_r-w)^{\epsilon/2} \xrightarrow{\text{B1}} \quad & \frac{1}{\epsilon}\delta(w)\,a^{3\epsilon/2}(1-a)^{-\epsilon/2}v^{1-2\epsilon}\frac{\Gamma(1+\epsilon)\,\Gamma(1+\epsilon/2)}{\Gamma(1+3\epsilon/2)}\,{}_2F_1\left[\epsilon,2\epsilon,1+\frac{3}{2}\epsilon;a\right] \\ & \times\left(\frac{(1-a)^{2\epsilon}}{2\epsilon}\delta(1-v)+\left((1-v)^{-1+2\epsilon}\right)_{+v[a,\underline{1}]}\right) \\ & +\left(w^{-1+\epsilon}\,(w-w_0)^{-1+\epsilon/2}\,(w_r-w)^{\epsilon/2}\right)_{+w[\underline{0},w_r]} \quad . \end{aligned} \tag{13}$$

Making this substitution in equation (12), we find that, in region B1, the leading singularity is a triple pole, proportional to $\delta(w)\,\delta(1-v)$.

Finally, in region B2, the factor $(1-u)^{-1}$ is also potentially divergent, and should be taken into account. In order to do that, we use the prescription given in equation (13) replacing a with z as the lower limit in the v prescription,

obtaining

$$\int_{x_u}^{1} du \int_{z}^{1} dv \int_{0}^{w_r} dw \, \frac{1}{\epsilon^2}\delta(w)\,(1-u)^{-1+\epsilon}\left(\frac{(1-z)^{2\epsilon}}{2\epsilon}\delta(1-v) + \left((1-v)^{-1+2\epsilon}\right)_{+v[z,\underline{1}]}\right)\tilde{f}(u,v,w,\epsilon)$$
$$+\int_{x_u}^{1} du \int_{z}^{1} dv \int_{0}^{w_r} dw \, \frac{1}{\epsilon}\,\frac{1}{1-u}\,\left(w^{-1+\epsilon}\,(w-w_0)^{-1+\epsilon/2}\,(w_r-w)^{\epsilon/2}\right)_{+w[\underline{0},w_r]}\, f(u,v,w,\epsilon)\,, \tag{14}$$

In the last equation we have factored out in $\tilde{f}(u,v,w,\epsilon)$ the expressions that are not relevant for the prescription. In the first term, the factor $(1-u)^{-1+\epsilon}$ can be prescribed with the usual recipe

$$(1-u)^{-1+\epsilon} \to \frac{1}{\epsilon}\,\delta(1-u) + \left((1-u)^{-1+\epsilon}\right)_{+u[0,\underline{1}]}\,, \tag{15}$$

whereas the second term can be safely expanded in power series of ϵ, since the u integral is convergent even when $\epsilon \to 0$. Indeed, in this case, the pole in $u=1$ is regulated by the fact that the w integration region shrinks to the point $w=0$ when $u \to 1$ whereas the integrand has no δ functions. The final result includes a quartic pole, as mentioned above, accompanied by $\delta(w)\delta(1-v)\delta(1-u)$.

All the terms singular along three intersecting curves found in the quark initiated corrections can be managed as in the previous example, and they present the distinctive feature of having singularities along $u=1$. Consequently, as it is shown above, the corresponding prescriptions have to be modified, but only in region B2. Given that the NLO non homogeneous kernels are determined by singularities along $w=1$ exclusively, associated to the prescription in the B0 region, and to terms which contribute in the forward direction in B1, they receive no contributions from the more involved triple overlapping singularities.

The virtual one loop contributions to both σ_{qg} and $\sigma_{q_i q_i}$ can be straightforwardly computed using the standard techniques [16] for handling the loop integrations. These corrections, with two partons in the final state, contribute to regions B1 and B2 but not to region B0. In this case, the singularities relevant for the renormalization of fracture functions, that is in the $w=1$ direction, are along the curve $v=a$ in region B1 and can be managed with the usual prescription recipes in one variable. On the other side, due to the one-body phase space of the outgoing quark, the two loop graphs in Figure 4 only have support in the point $u=v=1$, $w=0$ and thus their calculation is not required for the extraction of the non homogeneous kernels. The results for these contributions can be found in [17, 18, 19]. Of course, the complete calculation of the $\mathcal{O}(\alpha_s^2)$ cross sections requires to take into account not only the two loop quark form factor but also a full analysis of the singularities coming from the real and one loop virtual contributions, including regions B1 and B2.

III. FACTORIZATION OF SINGULARITIES AND EVOLUTION EQUATIONS

The order-α_s^2 one particle inclusive partonic cross sections described in the previous section, have a complex singularity structure, revealed as poles in ϵ. Poles associated with ultraviolet divergences are removed by coupling constant renormalization. On the other side, factorization theorems warrant that poles related to collinear singularities can be removed from the hadronic cross section by suitable redefinition of the bare parton densities, fragmentation and fracture functions in terms of renormalized ones. The self consistency of these factorization prescriptions fixes the scale dependence of the renormalized quantities, and in this case allows to obtain the NLO evolution kernels for the fracture functions.

In order to accomplish the factorization procedure, we begin writing the hadronic cross sections in terms of renormalized parton densities and fragmentation functions. Restricting our attention to the region B0, which contain all the forward singularities, we avoid the appearance of divergences at $u=1$ or $v=1$ as explained in Section II. Notice that in this way, we only have to take into account LO expressions for the bare parton densities and fragmentation functions, since NLO contributions, once convoluted with the tree level partonic cross section, give rise to terms with support in $u=1$ and $v=1$ which contribute only to regions B1 and B2.

As explained in reference [10], the convolutions between the LO kernels and the $\mathcal{O}(\alpha_s)$ cross sections can be better handled keeping to all orders in ϵ the expressions for the cross sections, and writing the subtracted 'plus' distributions in the kernels as

$$\left(\frac{1}{1-x}\right)_{+x[0,\underline{1}]} \to \lim_{\epsilon'\to 0}\,(1-x)^{-1+\epsilon'} - \frac{1}{\epsilon'}\,\delta(1-x) + \mathcal{O}(\epsilon')\,. \tag{16}$$

The limit $\epsilon' \to 0$ can be safely taken once the convolutions are explicitly evaluated. The resulting counterterms can then be subtracted from the $\mathcal{O}(\alpha_s^2)$ cross sections while ultraviolet singularities are removed by coupling constant

renormalization

$$\frac{\alpha_s}{2\pi} = \frac{\alpha_s\left(M_R^2\right)}{2\pi}\left(1+\frac{\alpha_s\left(M_R^2\right)}{2\pi}\, f_\Gamma\, \frac{\beta_0}{\epsilon}\left(\frac{M_R^2}{4\pi\mu^2}\right)^{\epsilon/2}\right). \tag{17}$$

β_0 is the lowest order coefficient function in the QCD β function,

$$\beta_0 = \frac{11}{3}\, C_A - \frac{4}{3}\, n_f T_F \tag{18}$$

where $C_A = N$ for $SU(N)$, $T_F = 1/2$ as usual, and n_f stands for the number of active quark flavors. M_R is the renormalization scale and

$$f_\Gamma = \frac{\Gamma(1+\epsilon/2)}{\Gamma(1+\epsilon)} \tag{19}$$

At this point, only poles proportional to $\delta(1-w)$ are left in region B0, to be factorized in the redefinition of the bare fracture functions. In terms of renormalized quantities, fracture functions can be written as

$$\begin{aligned} M_{i,h/P}(\xi,\zeta) \;=\; & \frac{1}{\xi}\int_{\xi}^{\frac{\xi}{\xi+\zeta}}\frac{du}{u}\int_{\frac{\zeta}{\xi}}^{\frac{1-u}{u}}\frac{dv}{v}\,\Delta_{ki\leftarrow j}(u,v,M_f)\, f^r_{j/P}\left(\frac{\xi}{u},M_f^2\right)\, D^r_{h/k}\left(\frac{\zeta}{\xi\, v},M_f^2\right) \\ & + \int_{\frac{\xi}{1-\zeta}}^{1}\frac{du}{u}\,\Delta_{i\leftarrow j}(u,M_f)\, M^r_{j,h/P}\left(\frac{\xi}{u},\zeta,M_f^2\right). \end{aligned} \tag{20}$$

where a sum over repeated indices is understood. Notice that we have chosen all the factorization scales to be equal to the renormalization scale.

Given that only single and double poles remain in the cross sections once that coupling constant, parton densities and fragmentations functions are renormalized, the functions $\Delta_{i\leftarrow j}$ and $\Delta_{ki\leftarrow j}$ can be written as

$$\Delta_{i\leftarrow j}(u,M_f^2) = \delta_{ij} - \frac{\alpha_s}{2\pi}\, f_\Gamma\left(\frac{M_f^2}{4\pi\mu^2}\right)^{\epsilon/2}\frac{2}{\epsilon}\, P^{(0)}_{i\leftarrow j}(u) + \left(\frac{\alpha_s}{2\pi}\right)^2 f_\Gamma^2\left(\frac{M_f^2}{4\pi\mu^2}\right)^{\epsilon}\left\{\frac{1}{\epsilon^2}R^{(1)}_{i\leftarrow j}(u) - \frac{1}{\epsilon}P^{(1)}_{i\leftarrow j}(u)\right\} \tag{21}$$

$$\Delta_{ki\leftarrow j}(u,v,M_f^2) = -\frac{\alpha_s}{2\pi}\, f_\Gamma\left(\frac{M_f^2}{4\pi\mu^2}\right)^{\epsilon/2}\frac{2}{\epsilon}\, \tilde{P}^{(0)}_{ki\leftarrow j}(u,v) + \left(\frac{\alpha_s}{2\pi}\right)^2 f_\Gamma^2\left(\frac{M_f^2}{4\pi\mu^2}\right)^{\epsilon}\left\{\frac{1}{\epsilon^2}R^{(1)}_{ki\leftarrow j}(u,v) - \frac{1}{\epsilon}P^{(1)}_{ki\leftarrow j}(u,v)\right\} \tag{22}$$

The $\Delta_{i\leftarrow j}$ functions in the homogeneous terms are just those found for parton densities in the inclusive case, and can be obtained from the corresponding transition functions in reference [11]. The LO non homogeneous kernels $\tilde{P}^{(0)}_{ki\leftarrow j}(u,v)$, can be written in terms of those in reference [2] as

$$\tilde{P}^{(0)}_{ki\leftarrow j}(u,v) = P^{(0)}_{ki\leftarrow j}(u)\,\delta\left(v - \frac{1-u}{u}\right). \tag{23}$$

Whereas the NLO kernels $P^{(1)}_{ki\leftarrow j}(u,v)$ can only be found after explicit calculation of the partonic cross sections up to order α_s^2, the requirement of consistency in the factorization process constraints the double pole coefficients $R^{(1)}_{ki\leftarrow j}(u,v)$. Indeed, in order to factorization be consistent, equation (20) has to be valid no matter the election of the factorization scale M_f and the corresponding renormalized quantities have to be well defined when $\epsilon \to 0$. These conditions imply that the $\mathcal{O}(\alpha_s^2)$ coefficients $R^{(1)}_{ki\leftarrow j}(u,v)$ are determined by the $\mathcal{O}(\alpha_s)$ kernels. Explicitly,

$$\begin{aligned} R^{(1)}_{ki\leftarrow j}(u,v) \;=\; & \sum_l \frac{2}{\epsilon^2}\left[\tilde{P}^{(0)}_{ki\leftarrow l}(u,v)\otimes P^{(0)}_{l\leftarrow j}(u) + \tilde{P}^{(0)}_{li\leftarrow j}(u,v)\otimes P^{(0)}_{k\leftarrow l}(v) + \tilde{P}^{(0)}_{kl\leftarrow j}(u,v)\otimes' P^{(0)}_{i\leftarrow l}(u)\right] \\ & + \frac{1}{\epsilon^2}\,\beta_0\, P^{(0)}_{ki\leftarrow j}(u,v), \end{aligned} \tag{24}$$

where the convolutions are defined as

$$f(u,v)\otimes g(u) \;=\; \int_u^{\frac{1}{1+v}}\frac{d\bar{u}}{\bar{u}}\, f(\bar{u},v)\, g\left(\frac{u}{\bar{u}}\right),$$

$$f(u,v)\otimes g(v) \;=\; \int_v^{\frac{1-u}{u}} \frac{d\bar{v}}{\bar{v}}\, f(u,\bar{v})\, g\left(\frac{v}{\bar{v}}\right)\,, \tag{25}$$
$$f(u,v)\otimes' g(u) \;=\; \int_u^{1-u\,v} \frac{d\bar{u}}{\bar{u}}\,\frac{u}{\bar{u}}\, f\left(\bar{u},\frac{u}{\bar{u}}v\right) g\left(\frac{u}{\bar{u}}\right)\,.$$

The fact that equation (24) actually holds for any combination of indices is a useful mean of checking the results and provides a non-trivial verification of factorization of collinear singularities.

Once the bare fracture functions in the hadronic cross section, equation (2), are written in terms of the renormalized quantities, equation (20), the explicit cancellation of forward collinear singularities, showing as single poles in the partonic cross sections, define the NLO evolution kernels for fracture functions.

Forward singularities coming from the σ_{qg} cross section determine the non homogeneous evolution kernel $P^{(1)}_{gq\leftarrow q}(u,v)$. Notice that the first index to the left in this notation refers to the parton that fragments into the final state hadron, the second labels the flavor of the fracture function that factorizes the corresponding pole, and the one to the right is for the initial parton. Due to charge conjugation symmetry $\sigma_{qg}=\sigma_{\bar{q}g}$ and thus $P^{(1)}_{gq\leftarrow q}(u,v)=P^{(1)}_{g\bar{q}\leftarrow \bar{q}}(u,v)$.

The kernels coming from the $\sigma_{q_i q_j}$ cross sections can be classified identifying the cases in which the quark that interacts with the photon carries either the flavor of the initial state or that of the quark that hadronizes, $P^{(1)}_{q_j q_i\leftarrow q_i}(u,v)$ and $P^{(1)}_{q_j \bar{q}_j\leftarrow q_i}(u,v)$ respectively. Notice that in the first case the corresponding pole in the cross section will be multiplied by e_i^2, in the second by e_j^2, while there are no singular contributions proportional $e_i\, e_j$. Charge conjugation implies:

$$\begin{aligned} P^{(1)}_{q_j q_i\leftarrow q_i} \;&=\; P^{(1)}_{q_j \bar{q}_i\leftarrow \bar{q}_i} \;=\; P^{(1)}_{\bar{q}_j q_i\leftarrow q_i} \;=\; P^{(1)}_{\bar{q}_j \bar{q}_i\leftarrow \bar{q}_i} \\ P^{(1)}_{q_j \bar{q}_j\leftarrow q_i} \;&=\; P^{(1)}_{\bar{q}_j q_j\leftarrow q_i} \;=\; P^{(1)}_{\bar{q}_j q_j\leftarrow \bar{q}_i} \;=\; P^{(1)}_{q_j \bar{q}_j\leftarrow \bar{q}_i} \end{aligned} \tag{26}$$

The cross sections $\sigma_{q_i q_i}$, with the same quark flavor in both the initial and final state, can also be classified according to the charge factor that accompanies the simple pole. For the case in which the charge factor is e_j^2 with $j\neq i$ (that is contributions coming from the first two diagrams in Figure 3), the factorization of forward singularities fixes the sum of $P^{(1)}_{q_i q_j\leftarrow q_i}(u,v)$ and $P^{(1)}_{q_i \bar{q}_j\leftarrow q_i}(u,v)$, but can not determine them individually. However, the symmetry under the exchange of the quark-antiquark pair of flavor j of the cross section, implies

$$P^{(1)}_{q_i q_j\leftarrow q_i} \;=\; P^{(1)}_{q_i \bar{q}_j\leftarrow \bar{q}_i}\,, \tag{27}$$

whereas charge conjugation gives

$$P^{(1)}_{\bar{q}_i q_j\leftarrow q_i} \;=\; P^{(1)}_{q_i q_j\leftarrow \bar{q}_i}\,. \tag{28}$$

Again there are no singularities in the $e_i e_j$ pieces.

The terms proportional to e_i^2 in the $\sigma_{q_i q_i}$ cross section contribute to the combination of the $P^{(1)}_{qq\leftarrow q}$ and $P^{(1)}_{q\bar{q}\leftarrow q}$ kernels. In this case, although charge conjugation implies

$$\begin{aligned} P^{(1)}_{qq\leftarrow q} \;&=\; P^{(1)}_{\bar{q}\bar{q}\leftarrow \bar{q}} \\ P^{(1)}_{q\bar{q}\leftarrow q} \;&=\; P^{(1)}_{\bar{q}q\leftarrow \bar{q}}\,, \end{aligned} \tag{29}$$

there is no symmetry at the cross section level relating $P^{(1)}_{qq\leftarrow q}$ and $P^{(1)}_{q\bar{q}\leftarrow q}$, and thus allowing to disentangle them. This means that the cancellation of the forward singularities in this particular process would not allow to obtain the scale dependence of the valence non-singlet combinations of quark fracture functions.

On the other hand, singlet combinations of fracture functions, and also those non-singlet combinations that include $M_{q,h/P}+M_{\bar{q},h/P}$, evolve with precisely the above mentioned combination of kernels. Notice that these last two combinations are the ones that actually occur in the hadronic cross sections, thus, although the kernels obtained so far are not enough to perform the evolution of individual flavors, they suffice to obtain the scale dependence of the cross section.

Finally, the simple pole in the $\sigma_{q\bar{q}}$ cross section is factorized into the $P^{(1)}_{\bar{q}q\leftarrow q}$ kernel, which due to charge conjugation satisfies:

$$P^{(1)}_{\bar{q}q\leftarrow q} \;=\; P^{(1)}_{q\bar{q}\leftarrow \bar{q}}\,. \tag{30}$$

Explicit expressions for the above mentioned kernels are given in Appendix B.

Having computed the relevant kernels, the evolution equations for the corresponding fracture function flavor combinations can be obtained taking moments in two variables of their respective factorization prescriptions, equation (20), as it is done in reference [10]. Taking the derivative with respect to the scale M^2, the resulting algebraic equations for the fracture functions moments, can then be solved using the already known explicit solutions for those of the parton densities and fragmentation functions. Inverting moments and using eqs.(17-20) we finally arrive to:

$$\frac{\partial\, M^r_{i,h/P}(\xi,\zeta,M^2)}{\partial \log M^2} = \frac{\alpha_s(M^2)}{2\pi}\int_{\frac{\xi}{1-\zeta}}^{1} \frac{du}{u}\,\left[P^{(0)}_{i\leftarrow j}(u) + \frac{\alpha_s(M^2)}{2\pi}\,P^{(1)}_{i\leftarrow j}(u)\right]\,M^r_{j,h/P}\left(\frac{\xi}{u},\zeta,M^2\right)$$
$$+\frac{\alpha_s(M^2)}{2\pi}\,\frac{1}{\xi}\int_{\xi}^{\frac{\xi}{\xi+\zeta}}\frac{du}{u}\int_{\frac{\zeta}{\xi}}^{\frac{1-u}{u}}\frac{dv}{v}\,\left[\tilde{P}^{(0)}_{ki\leftarrow j}(u,v) + \frac{\alpha_s(M^2)}{2\pi}\,P^{(1)}_{ki\leftarrow j}(u,v)\right]\,f^r_{j/P}\left(\frac{\xi}{u},M^2\right)\,D^r_{h/k}\left(\frac{\zeta}{\xi\, v},M^2\right)\,, \quad (31)$$

where, again, we have chosen all the scales to be equal. At order α_s, the Dirac delta of the non homogeneous kernels, equation (23), allows to trivially perform the integration over the v variable for the non homogeneous terms in equation (31), which can be recasted in the more familiar form of reference [1]. However, at NLO the kernels $P^{(1)}_{ki\leftarrow j}(u,v)$ are no longer constrained to the curve $v = (1-u)/u$ and thus the general expressions for the evolution equations involve a double convolution.

IV. PHENOMENOLOGY

In this section we evaluate the phenomenological consequences of the non homogeneous corrections computed so far, examining the impact of these corrections relative to the homogeneous evolution [20]. Unfortunately, there is scarce direct information on semi-inclusive cross sections with a wide kinematical coverage, as required as input for the evolution equations. However there are sensible model estimates available, which can used for the present purpose, in particular for π^+ production from proton targets.

In order to implement the comparison, we consider a model estimate of the corresponding fracture function $M_{q,\pi^+/P}$ based on the ideas of reference [21] for the description of forward hadrons in DIS, already applied to fracture functions in reference[9]. In this approach, the fracture function is written as

$$M_{q,\pi^+/P}(x_B,x_L,Q_0^2), \simeq \phi_{\pi^+/p}(x_L) q(x_B,Q_0^2)\,, \quad (32)$$

in terms of the flux of positive pions in the proton times the quark densities in a neutron. x_L is the ratio between the final state hadron energy and that of the proton target in the laboratory frame. For very forward hadrons, $x_L \sim (1-x_B)\,v_h$. Taking this estimate as an adequate approximation for $M_{q,\pi^+/P}$ at given initial scale Q_0^2, it is possible to obtain the fracture densities at higher scales using the evolution equations obtained in the last section, and realistic parameterizations for parton densities [22] and fragmentation functions [23].

As we have anticipated, although the results obtained in the previous section, together with those of reference [10] for the gluon initiated contributions, complete the QCD corrections up to order-α_s^2 in the cross sections and thus the NLO kernels required for M_q, we still do not know those required for the evolution of the gluon fracture density M_g. These kernels show up in the order-α_s^3 cross section and contribute indirectly to the evolution of M_q through the homogeneous terms in the evolution equations, meanwhile it is worth assessing the effects of its direct NLO contributions.

In Figure 5 we plot the outcome of evolving the model estimate for $M_{q,\pi^+/P}$ from an initial scale $Q_0^2 = 1\,GeV^2$ to $Q^2 = 100\,GeV^2$ as a function of x_B for four different values of x_L. The solid lines correspond to the full NLO evolution for the flavor singlet combination of $M_{q,\pi^+/P}$

$$M^{\pi^+/P}_{singlet} \equiv \sum_{q,\bar{q}} M_{q,\pi^+/P}, \quad (33)$$

the dashed lines come from using LO evolution, and the dots show the evolution using only homogeneous terms.

NLO non homogeneous corrections clearly have a different relative impact depending on the kinematical region analysed. For example, in the low x_L region, they dominate over the homogeneous terms and their effects become as large as those of the LO terms for large x_B. In this region the difference between LO and NLO evolution is almost independent of x_B, however, the LO non homogeneous evolution effects increase considerably as x_B falls. In Figure 6 we show the same as in Figure 5 but as a function of Q^2 for $x_B = 0.15$.

For larger fractions of hadron momentum, both the LO and NLO non homogeneous contributions become more and more suppressed, and although NLO effects overpower LO corrections, they can not match the growth of the

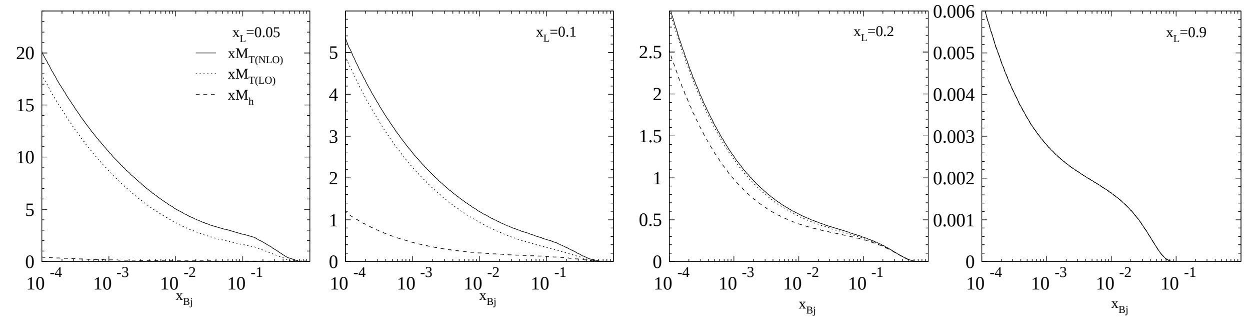

FIG. 5: Flavor Singlet combination of xM_q at $Q^2 = 100\,GeV^2$ as evolved from $Q_0^2 = 1\,GeV^2$ including NLO non homogeneus corrections for different values of the pion energy fraction x_L. Solid lines correspond to the full NLO evolution, dashed lines correspond to LO evolution and dotted lines show the evolution using only homogeneous terms.

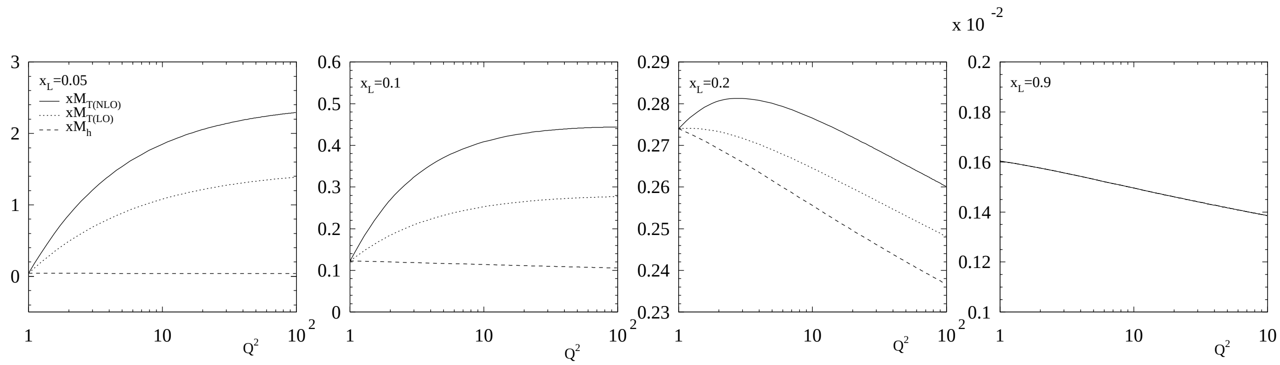

FIG. 6: Scale dependence of xM_q for $x_B = 0.15$

homogeneous contributions. Large hadron momentum fraction is precisely the region relevant for hard diffractive interactions. The lack of significant non homogeneus effects in the evolution in this region, is consistent with previous studies on the scale dependence of diffractive deep inelastic data [6].

Notice that the relative importance of homogeneous and non homogeneous contributions to the evolution depends crucially on the the size and shape of the appropriate fragmentation and fracture functions corresponding to the process considered. This means that the specific energy fraction at which non homogeneus effects become non negligible is characteristic of each process,and in this particular case, depend on the model estimate used for the fracture function, as well as on the parameterizations used for parton densities and fragmentation functions. From another side, the value chosen in the present estimate as departing point for the evolution, i.e. the Q^2 value Q_0^2 at which we assume the model estimate to hold, modifies to some extent the outcome of the evolution and thus the onset of non homogeneous effects. In any case, the numerical estimates shown in this section illustrate the sort of scale dependence one can expect from a given one particle inclusive processes in the light of our present knowledge of QCD corrections.

V. CONCLUSIONS

We have computed the $\mathcal{O}(\alpha_s^2)$ quark initiated corrections to one particle inclusive deep inelastic scattering cross sections, verifying and extending for these processes the feasibility of the approach proposed in reference [10] to deal with overlapping singularities. Although quark initiated corrections include a more complex singularity structure, once backward configurations are regulated, forward singularities can be treated in a similar way to those in gluon initiated corrections.

We made an explicit check of the factorization of collinear singularities arising from quark initiated corrections at this order and obtained the evolution kernels required to compute non homogeneous contributions to the scale dependence. These kernels, which are labeled by three indices, corresponding to the nature of the parton that interacts with photon, hadronizes, or initiates the hard processes, respectively, show up entangled in the cross section and can not be extracted individually for each flavor from SIDIS. However, flavor combinations required for the singlet

and total flavor combinations can be obtained. Finally, using a model estimate for fracture functions of protons into pions, we found that the impact in the scale dependence of non homogeneous NLO corrections relative to LO and to homogeneous terms depends strongly on the kinematical region under analysis. Specifically, NLO corrections are particularly large for small values of parton and pion momentum fractions.

VI. ACKNOWLEDGEMENTS

We warmly acknowledge C. A. García Canal, D. de Florian, and G. Félix for comments and suggestions.

Appendix A

In this appendix we present the angular integrals required in the computation of the quark initiated corrections to SIDIS at order $\mathcal{O}(\alpha_s^2)$. As usual, all these integrals can be put, by means of partial fractioning, in the standard form [15]

$$I(k,l) = \int_0^\pi d\beta_1 \int_0^\pi d\beta_2 \, \frac{\sin^{1+\epsilon}\beta_1 \sin^\epsilon \beta_2}{(a+b\cos\beta_1)^k (A+B\cos\beta_1 + C\sin\beta_1\cos\beta_2)^l} \,, \tag{34}$$

and can be classified according to whether their parameters satisfy either $a^2 = b^2$ or $A^2 = B^2 + C^2$, both relations simultaneously, or neither of them. In the following table, these cases are labeled as $I_a(k,l)$, $I_A(k,l)$, $I_{Aa}(k,l)$ and $I_0(k,l)$ respectively. The parameters a, b, A, B and C, are functions of the variables x_B, u, v and w. The resulting integrals as well as their coefficients can develop further singularities associated to these variables, what requires some care when expanding in power series of the regulator ϵ. Specifically, the limit $w \to 1$ implies always that $b \to \pm a$, $C \to 0$ and $B \to \pm A$, which means that factors like $(A\,b - a\,B)^\epsilon$ or $(a^2-b^2)^\epsilon$ often present in the expressions have to be kept to all orders. This factorization is trivial for the integrals that can be computed to all orders in ϵ, but as it was mentioned in Section II, some special care is required for the integrals that have to be expanded before calculation. All the integrals of this last type can be handled with the method described in reference [10].

The integrals that can be done to all orders in ϵ are:

- I_0: $a^2 \neq b^2$ and $A^2 \neq B^2 + C^2$.

$$I_0(0,0) = \frac{2\pi}{1+\epsilon} \,, \tag{35}$$

$$I_0(-1,0) = a\, I_0(0,0) \,, \tag{36}$$

$$I_0(-1,-1) = \left[a\,A + \frac{b\,B}{3+\epsilon} \right] I_0(0,0) \,, \tag{37}$$

$$I_0(-1,-2) = \left[a\,A^2 + \frac{2\,A\,B\,b + a\,(B^2+C^2)}{3+\epsilon} \right] I_0(0,0) \,, \tag{38}$$

$$I_0(-2,0) = \left[a^2 + \frac{b^2}{3+\epsilon} \right] I_0(0,0) \,, \tag{39}$$

$$I_0(-2,-1) = \left[a^2\,A + \frac{A\,b^2 + 2\,a\,b\,B}{3+\epsilon} \right] I_0(0,0) \,, \tag{40}$$

$$I_0(-2,-2) = \left[a^2\,A^2 + \frac{A^2\,b^2 + 4\,A\,a\,B\,b + a^2\,(B^2+C^2)}{3+\epsilon} + \frac{b^2\,(3\,B^2+C^2)}{(3+\epsilon)(5+\epsilon)} \right] I_0(0,0) \,, \tag{41}$$

$$I_0(-3,0) = \left[a^3 + \frac{3\,a\,b^2}{3+\epsilon} \right] I_0(0,0) \,, \tag{42}$$

$$I_0(-3,-1) = \left[a^3\,A + \frac{3\,a\,b\,(A\,b + a\,B)}{3+\epsilon} + \frac{3\,b^3\,B}{(3+\epsilon)(5+\epsilon)} \right] I_0(0,0) \,, \tag{43}$$

$$I_0(1,0) = \frac{2\pi}{b\,\epsilon} \left\{ 1 - (a+b)^{-\epsilon} \left(a^2-b^2\right)^{\epsilon/2} {}_2F_1\left[\frac{\epsilon}{2}, \epsilon, 1+\epsilon; \frac{2b}{a+b} \right] \right\} \,, \tag{44}$$

$$I_0(1,-1) = \frac{A\,b - a\,B}{b}\, I_0(1,0) + \frac{2\pi\,B}{(1+\epsilon)\,b} \,, \tag{45}$$

$$I_0(1,-2) = \frac{1}{b^2}\left[(A\,b - a\,B)^2 - \frac{C^2(a^2-b^2)}{2+\epsilon}\right] I_0(1,0) + \frac{2\pi}{(1+\epsilon)\,b^2}\left[2A\,B\,b - a\,B^2 + \frac{a\,C^2}{2+\epsilon}\right], \tag{46}$$

$$\begin{aligned} I_0(1,-3) = {} & \frac{1}{b^3}\left[(A\,b-a\,B)^3 - \frac{3\,(A\,b-a\,B)(a^2-b^2)\,C^2}{2+\epsilon}\right] I_0(1,0) \\ & + \frac{2\pi}{(1+\epsilon)\,b^3}\left[B\,(3\,A\,b\,(A\,b-a\,B)+a^2B^2) + \frac{b^2\,B^3}{3+\epsilon} + 3\,C^2\left(\frac{B\,b^2}{3+\epsilon} + \frac{a\,(A\,b-a\,B)}{2+\epsilon}\right)\right], \end{aligned} \tag{47}$$

$$\begin{aligned} I_0(1,-4) = {} & \frac{1}{b^4}\left[(A\,b-a\,B)^4 - \frac{6\,(a\,b-a\,B)^2(a^2-b^2)\,C^2}{2+\epsilon} + \frac{3\,(a^2-b^2)^2\,C^2}{(2+\epsilon)(4+\epsilon)}\right] I_0(1,0) \\ & + \frac{2\pi}{(1+\epsilon)\,b^4}\Bigg[4\,A\,B\,b\,(A\,b-a\,B)^2 + a\,B^2\,(2\,A^2\,b^2 - a^2\,B^2) + \frac{(4\,A\,B\,b - a\,B^3)\,b^2\,B}{3+\epsilon} \\ & + \frac{6\,C^2\,b^2\,B\,(A\,b-a\,B)}{3+\epsilon} + \frac{6\,C^2\,a\,(A\,b-a\,B)^2}{2+\epsilon} - \frac{3\,a\,C^4\,(a^2-b^2}{(2+\epsilon)(3+\epsilon)} + \frac{3\,a\,b\,C^2}{(3+\epsilon)(4+\epsilon)}\Bigg], \end{aligned} \tag{48}$$

$$I_0(2,0) = -\frac{a\,\epsilon}{a^2-b^2}\,I_0(1,0) + \frac{2\pi}{a^2-b^2}, \tag{49}$$

$$I_0(2,-1) = \frac{1}{\epsilon\,a\,b}\left[a\,(A\,b-a\,B)\,\epsilon - B\,(a^2-b^2)\right] I_0(2,0) + \frac{2\pi\,B}{\epsilon\,a\,b}, \tag{50}$$

$$\begin{aligned} I_0(2,-2) = {} & \frac{1}{\epsilon\,a\,b^2}\left[a\,(A\,b-a\,B)^2\epsilon - 2\,B\,(a^2-b^2)\,(A\,b-a\,B) - a\,C^2\,(a^2-b^2)\right] I_0(2,0) \\ & + \frac{2\pi}{\epsilon\,a\,b^2}\left[B\,(2\,A\,b-a\,B) + \frac{a\,(C^2-B^2)}{1+\epsilon}\right], \end{aligned} \tag{51}$$

$$\begin{aligned} I_0(2,-3) = {} & \frac{1}{\epsilon\,a\,b^3}\Bigg[a\,(A\,b-a\,B)^3\epsilon - 3\,B\,(a^2-b^2)\,(A\,b-a\,B)^2 - 3\,C^2\,(a^2-b^2) \\ & \times\left(a\,(A\,b-a\,B) - \frac{B\,(a^2-b^2)}{2+\epsilon}\right)\Bigg] I_0(2,0) + \frac{2\pi}{\epsilon\,a\,b^3}\Bigg[3\,C^2\,\left(\frac{b^2\,B}{2+\epsilon} + \frac{a\,(A\,b-a\,B)}{1+\epsilon} - \frac{a^2\,B}{(1+\epsilon)(2+\epsilon)}\right) \\ & + B\,\left(3\,A\,b\,(A\,b-a\,B) + a^2\,B^2 + \frac{2\,a^2\,B^2 - 3\,a\,A\,b\,B}{1+\epsilon}\right)\Bigg], \end{aligned} \tag{52}$$

where $a > b > 0$. Notice that the cases $a^2 = b^2$ and $A^2 = B^2 + C^2$ corresponding to integrals (36)-(43) can be easily obtained taking the appropriate limit, whereas for integrals (44)-(52) only the limit $C^2 \to A^2 - B^2$ can be taken, giving the corresponding I_A integral. The hypergeometric function that appears in the cases $k > 0$ can be expanded in power series in ϵ [24]:

$$\begin{aligned} {}_2F_1\left[\frac{\epsilon}{2},\epsilon,1+\epsilon;z\right] = {} & 1 + \frac{\epsilon^2}{2}\,Li_2(z) + \frac{\epsilon^3}{8}\big(\log^2(1-z)\log(z) + 2\,\log(1-z)\,\mathrm{Li}_2(1-z) \\ & - 2\,\mathrm{Li}_3(1-z) + 2\,\zeta(3) - 4\,\mathrm{Li}_3(z)\big) + \mathcal{O}(\epsilon^4) \end{aligned} \tag{53}$$

- I_{Aa}: $a^2 = b^2$, $A^2 - B^2 - C^2 = 0$. Using the results in [25], all these integrals can be put in the standard form [24]:

$$I_{Aa}(k,l) = \frac{2\pi}{a^k\,A^l}\,a^{-k-l}\frac{\Gamma(1+\epsilon)}{\Gamma(1+\frac{\epsilon}{2})^2}\,B\left(1-k+\frac{\epsilon}{2},1-l+\frac{\epsilon}{2}\right)\,{}_2F_1\left[k,l,1+\frac{\epsilon}{2};\frac{A-B}{2\,A}\right], \tag{54}$$

where $0 \le -\frac{C}{A}, \frac{B}{A} \le 1$. The hypergeometric functions appearing in the cases $k, l > 0$ should be carefully treated as they usually develop singularities in the limit $B \to -A$. Using the relations in sections (2.8) and (2.9) in [26] the above mentioned hypergeometric functions can be written always in terms of

$${}_2F_1\left[\frac{\epsilon}{2},\frac{\epsilon}{2},1+\frac{\epsilon}{2};\frac{A-B}{2\,A}\right] \tag{55}$$

which is regular when $B = -A$ and admits the expansion [24]:

$${}_2F_1\left[\frac{\epsilon}{2},\frac{\epsilon}{2},1+\frac{\epsilon}{2};z\right] = 1 + \frac{\epsilon^2}{4}\,Li_2(z) + \frac{\epsilon^3}{16}\big(\log^2(1-z)\log(z) + 2\,\log(1-z)\,\mathrm{Li}_2(1-z)$$

$$- 2\,\mathrm{Li}_3(1-z) + 2\,\zeta(3) - 2\,\mathrm{Li}_3(z)\Big) + \mathcal{O}(\epsilon^4) \tag{56}$$

All the integrals which cannot be calculated to all orders in ϵ are of the classes $I_A(k,l)$ or $I_a(k,l)$ with $k,l>0$. With an adequate choice of the reference frame, the latter integrals can be recasted in terms of $I_A(k,l)$ integrals also, which are considerably simpler to calculate. The 10 resulting integrals belong to the sets $I_A(1,1)$, $I_A(2,1)$, $I_A(1,2)$ and $I_A(2,2)$. In each case, the potentially singular factors have to be extracted, as described in [10], before the expansion in powers of ϵ. With the exception of two integrals of the type $I_A(1,1)$, all the integrals present potential singularities due to a factor $(A\,a-b\,B)^{-n+\epsilon}$ with $n=1,2,3$. In the remaining two, corresponding to the $I_A(1,1)$ case, a factor $(a^2-b^2)^{\epsilon/2}$ has to be kept to all orders. Explicitly:

$$\begin{aligned} I_A(1,1) &= \pi\,(A\,a-b\,B)^{-1+\epsilon/2}\left\{\frac{2}{\epsilon}\,\frac{\Gamma(1+\epsilon)}{\Gamma(1+\frac{1}{\epsilon})^2}\,A^{-\epsilon/2}(a-b)^{\epsilon/2}{}_2F_1\left[\frac{\epsilon}{2},\frac{\epsilon}{2},1+\frac{\epsilon}{2};-\frac{b\,(A-B)}{A\,(a-b)}\right] - \log\left(\frac{A\,(a+b)}{A\,a-b\,B}\right)\right. \\ &- \frac{\epsilon}{2}\left[\log\left(\frac{a+b}{a-b}\right)\,\log\left(-\frac{b\,(A-B)}{A\,(a-b)}\right) + \frac{1}{2}\,\log\left(\frac{A\,(a+b)}{A\,a-b\,B}\right)\,\log\left(\frac{A\,(a+b)}{b^2\,(A-B)^2\,(A\,a-b\,B)}\right)\right. \\ &\left.\left. + \frac{1}{2}\,\mathrm{Li}_2\left(-\frac{b\,(A+B)}{A\,a-b\,B}\right) + \mathrm{Li}_2\left(\frac{A\,a-b\,B}{A\,(a-b)}\right)\right] + \mathcal{O}(\epsilon^2)\right\},\ \text{case 1} \end{aligned} \tag{57}$$

$$\begin{aligned} I_A(1,1) &= \frac{4\pi}{\epsilon}\,A^{-\epsilon/2}\,(a-b)^{-\epsilon/2}\,(A\,a-b\,B)^{-1+\epsilon/2}\,{}_2F_1\left[\frac{\epsilon}{2},\frac{\epsilon}{2},1+\frac{\epsilon}{2};-\frac{b\,(A-B)}{A\,(a-b)}\right] - \frac{\pi\,(a^2-b^2)^{\epsilon/2}}{A\,a-b\,B}\left\{\frac{2}{\epsilon}\,A^{\epsilon/2}\right. \\ &\times (a-b)^{-\epsilon/2}\,(A\,a-b\,B)^{-\epsilon/2}\,{}_2F_1\left[\frac{\epsilon}{2},\epsilon,1+\epsilon;-\frac{b\,(A+B)}{A\,a-b\,B}\right] - \log\left(\frac{A\,(a-b)}{A\,a-b\,B}\right) \\ &\left. + \frac{\epsilon}{4}\left[\log^2\left(A\,(a-b)\right) - 2\,\log\left(\frac{A}{a-b}\right)\,\log\left(\frac{A\,(a-b)}{A\,a-b\,B}\right) - \log^2\left(A\,a-b\,B\right)\right] + \mathcal{O}(\epsilon^2)\right\},\ \text{case 2} \end{aligned} \tag{58}$$

$$\begin{aligned} I_A(2,1) &= \frac{\pi}{A-B}\,(a+b)^{-1+\epsilon/2}\left\{(A+B)\,A^{\epsilon/2}\,(A\,a-b\,B)^{-1-\epsilon/2}\,{}_2F_1\left[\frac{\epsilon}{2},\epsilon,1+\epsilon;-\frac{b\,(A+B)}{A\,a-b\,B}\right]\right. \\ &\left. - 2\,(a-b)^{-1-\epsilon/2}\,{}_2F_1\left[\frac{\epsilon}{2},\epsilon,1+\epsilon;-\frac{2\,b}{a-b}\right]\right\} + \frac{2\pi}{(a-b)\,(A\,a-b\,B)} + \frac{\pi}{\epsilon}\,\frac{\Gamma(1+\epsilon)}{\Gamma\left(1+\frac{\epsilon}{2}\right)^2}\,A^{1-\epsilon/2}\,(a-b)^{-\epsilon/2} \\ &\times (A\,a-b\,B)^{-2+\epsilon/2}\,(2-\epsilon)\,{}_2F_1\left[\frac{\epsilon}{2},\frac{\epsilon}{2},1+\frac{\epsilon}{2};-\frac{2\,b}{a-b}\right]\,{}_2F_1\left[\frac{\epsilon}{2},\frac{\epsilon}{2},1+\frac{\epsilon}{2};-\frac{b\,(A-B)}{A\,(a-b)}\right] \\ &- \pi\,(A\,a-b\,B)^{-2+\epsilon/2}\left\{\frac{b\,(A+B)}{a+b} + A\,\log\left(\frac{A\,(a+b)}{A\,a-b\,B}\right) + \frac{\epsilon}{2}\left[\frac{A\,(a-b)\,(A+B)}{(a+b)\,(A-B)}\,\log(a-b)\right.\right. \\ &- \frac{b\,(A+B)}{a+b}\,\log(A) - \frac{(A+B)\,(A\,a-b\,B)}{(a+b)\,(A-B)}\,\log(a+b) + A\,\log\left(\frac{a+b}{a-b}\right)\,\log\left(-\frac{b\,(A-B)}{A\,(a-b)}\right) \\ &- \frac{2\,(A\,(A\,b-a\,B)+(A-B)\,(A\,a-b\,B))}{(a+b)\,(A-B)}\,\log\left(\frac{A\,(a+b)}{A\,a-b\,B}\right) \\ &+ \frac{A}{2}\,\log\left(\frac{A\,(a+b)}{A\,a-b\,B}\right)\,\log\left(\frac{A\,(a+b)}{b^2\,(A-B)^2\,(A\,a-b\,B)}\right) + A\left(\mathrm{Li}_2\left(\frac{A\,a-b\,B}{A\,(a-b)}\right) + 2\,\mathrm{Li}_2\left(-\frac{b\,(A+B)}{A\,a-b\,B}\right)\right. \\ &\left.\left.\left. - \mathrm{Li}_2\left(-\frac{2\,b}{a-b}\right)\right)\right] + \mathcal{O}(\epsilon^2)\right\}, \end{aligned} \tag{59}$$

$$\begin{aligned} I_A(1,2) &= -\frac{\pi}{(A\,a-b\,B)^2}\left\{\frac{\Gamma(1+\epsilon)}{4\,\Gamma(1+\frac{1}{\epsilon})^2}\left[(a+b)(2-\epsilon) + \frac{1}{\epsilon}\,A^{-\epsilon/2}\,(a-b)^{-\epsilon/2}(A\,a-b\,B)^{\epsilon/2}\,\Bigg(2\,(a+b)\,\epsilon - 8\,b\right.\right. \\ &\left. + \frac{b\,(a-b)\,(A+B)\,(2-\epsilon)\,(4-\epsilon)}{A\,a-b\,B}\Bigg)\,{}_2F_1\left[\frac{\epsilon}{2},\frac{\epsilon}{2},1+\frac{\epsilon}{2};-\frac{b\,(A-B)}{A\,(a-b)}\right]\right] + \frac{1}{A}\left[2\,b\,B - A\,(a-b)\right. \\ &\left. + \frac{2\,(A\,a-b\,B)}{2-\epsilon} - \frac{1}{2}\,b\,(A+B)A^{\epsilon/2}\,(a+b)^{\epsilon/2}(A\,a-b\,B)^{-\epsilon/2}\,{}_2F_1\left[\frac{\epsilon}{2},\epsilon,1+\epsilon;-\frac{b\,(A+B)}{A\,a-b\,B}\right]\right] \end{aligned}$$

$$
\begin{aligned}
&+ (A\,a - b\,B)^{\epsilon/2} \Bigg[\frac{b^2\,(A^2-B^2)}{2\,A\,(A\,a-b\,B)} + \frac{b\,(A\,b-a\,B)}{A\,a-b\,B} \log\left(\frac{A\,(a+b)}{A\,a-b\,B}\right) + \epsilon \Bigg[\frac{A\,(a^2+b^2) - 2\,a\,b\,B}{4\,(A\,a-b\,B)} \\
&- \frac{b^2\,(A^2-B^2)}{4\,A\,(A\,a-b\,B)} \log\left(A\,(a-b)\right) - \frac{A\,a+2\,A\,b+B\,b}{4\,A} \log\left(\frac{a+b}{a-b}\right) - \frac{b\,(A\,b-a\,B)}{A\,a-b\,B} \log\left(\frac{A\,(a+b)}{A\,a-b\,B}\right) \\
&+ \frac{A\,(a-b)+2\,B\,b}{4\,A} \log\left(\frac{A\,(a+b)}{A\,a-b\,B}\right) + \frac{b\,(A\,b-a\,B)}{2\,(A\,a-b\,B)} \log\left(\frac{a+b}{a-b}\right) \log\left(-\frac{b\,(A-B)}{A\,(a-b)}\right) \\
&+ \frac{b\,(A\,b-a\,B)}{4\,(A\,a-b\,B)} \log\left(\frac{A\,(a+b)}{A\,a-b\,B}\right) \log\left(\frac{A\,(a+b)}{b^2\,(A-B)^2\,(A\,a-b\,B)}\right) + \frac{b\,(A\,b-a\,B)}{2(A\,a-b\,B)} \left(\mathrm{Li}_2\left(-\frac{b\,(A+B)}{A\,a-b\,B}\right)\right. \\
&\left.+ \mathrm{Li}_2\left(\frac{A\,a-b\,B}{A\,(a-b)}\right)\right)\Bigg] + \mathcal{O}(\epsilon^2)\Bigg]\Bigg\}, \qquad (60)
\end{aligned}
$$

$$
\begin{aligned}
I_A(2,2) = &\frac{\pi}{(A\,a-b\,B)^3} \Bigg\{ \frac{\Gamma(1+\epsilon)}{8\,\Gamma\left(1+\frac{\epsilon}{2}\right)^2} \Bigg[-\frac{(2-\epsilon)\,(a+b)}{a-b}\,\left(A\,(a-b)\,(4-\epsilon) - 2\,(A\,a-b\,B)\right) + \frac{1}{\epsilon}\,A^{-\epsilon/2}\,(a-b)^{-\epsilon/2} \\
&\times (A\,a-b\,B)^{-1+\epsilon/2} \Big(4\,A^2\,b^2\,(3-\epsilon)\,(4-\epsilon) - 2\,\epsilon\,(2-\epsilon)\,(A\,a+b\,B)^2 + A\,b\,(A\,(a-b) - B\,(a+b))\,\epsilon\,(4-\epsilon)^2 \\
&+ 2\,A\,a\,B\,b\,(2-\epsilon)\,\epsilon^2 - 4B\,b\,(1-\epsilon)\,(4-\epsilon)\,(A\,a\,(2-\epsilon)+B\,b) \Big)\, {}_2F_1\left[\frac{\epsilon}{2},\frac{\epsilon}{2},1+\frac{\epsilon}{2};-\frac{b\,(A-B)}{A\,(a-b)}\right] \Bigg] \\
&+ \frac{4-\epsilon}{8} \Bigg[4\,(A\,(a+b) - 3\,b\,(A+B)) - \frac{8\,(A\,a-b\,B)}{2-\epsilon} + 2\,b\,(A+B)\,\epsilon + \frac{b\,(A+B)}{A\,(a+b)} \\
&\times A^{\epsilon/2}\,(a+b)^{\epsilon/2} (A\,a-b\,B)^{-\epsilon/2}\,(2\,A\,(a+b)\,(2-\epsilon) + b\,(A+B)\,\epsilon) {}_2F_1\left[\frac{\epsilon}{2},\epsilon,1+\epsilon;-\frac{b\,(A+B)}{A\,a-b\,B}\right] \Bigg] \\
&+ (A\,a-b\,B)^{\epsilon/2} \Bigg[-\frac{b^2\,(A+B)\,(5\,(A\,b-a\,B)+3\,(A\,a-b\,B)}{2\,(a+b)\,(A\,a-b\,B)} + \frac{b\,(3\,A\,(A\,b-a\,B)+B\,(A\,a-b\,B))}{A\,a-b\,B} \\
&\times \log\left(\frac{A\,(a+b)}{A\,a-b\,B}\right) + \epsilon \Bigg[\frac{1}{4} \left(A\,(a-b) - \frac{2\,b^2\,(A-B)}{a-b} + \frac{2\,b^2\,(A+B)^2}{A\,(a+b)} - \frac{2\,b^2\,(A^2-B^2)}{A\,a-b\,B} \right) \\
&+ \frac{b^2\,(A+B)\,(5\,(A\,b-a\,B)+3\,(A\,a-b\,B)}{4\,(a+b)\,(A\,a-b\,B)} \log\left(A\,(a-b)\right) - \frac{6\,b\,(A\,b-a\,B) - (a+3\,b)\,(A\,a-b\,B)}{4\,(a-b)} \\
&\times \log\left(\frac{a+b}{a-b}\right) + \frac{(A\,a-b\,B)\,(A\,a+2\,A\,b+b\,B) - 16\,A\,b\,(A\,b-a\,B)}{4\,(A\,a-b\,B)} \log\left(\frac{A\,(a+b)}{A\,a-b\,B}\right) \\
&+ \frac{b\,(A\,a-b\,B+2\,(A\,b-a\,B)}{2\,(a+b)} \log\left(\frac{A\,(a+b)}{A\,a-b\,B}\right) + \frac{b\,(3\,A\,(A\,b-a\,B)+B\,(A\,a-b\,B))}{2\,(A\,a-b\,B)} \log\left(\frac{a+b}{a-b}\right) \\
&\times \log\left(-\frac{b\,(A-B)}{A\,(a-b)}\right) + \frac{b\,(3\,A\,(A\,b-a\,B)+B\,(A\,a-b\,B))}{4\,(A\,a-b\,B)} \log\left(\frac{A\,(a+b)}{A\,a-b\,B}\right) \log\left(\frac{A\,(a+b)}{b^2\,(A-B)^2\,(A\,a-b\,B)}\right) \\
&+ \frac{b\,(3\,A\,(A\,b-a\,B)+B\,(A\,a-b\,B))}{2(A\,a-b\,B)} \left(\mathrm{Li}_2\left(-\frac{b\,(A+B)}{A\,a-b\,B}\right) + \mathrm{Li}_2\left(\frac{A\,a-b\,B}{A\,(a-b)}\right)\right)\Bigg] + \mathcal{O}(\epsilon^2)\Bigg]\Bigg\}. \qquad (61)
\end{aligned}
$$

(62)

For the $I_A(1,1)$ integrals, case 1 means that in the limit $w \to 1$ the parameters satisfy $b \to -a$ and $B \to -A$, whereas in case 2 they satisfy $b \to -a$ and $B \to A$. In this last case the factors $(A\,a - b\,B)^{-n}$ remain finite when $w \to 1$.

Notice that in the above list, there are some integrals that can be obtained from another by a suitable transformation, for example, integral $I_0(2,0)$ can be obtained from $I_0(1,0)$ by derivation with respect to parameter a. Further relations can be obtained using the results presented in [24].

Appendix B

In this appendix we present results for the NLO non homogeneous kernels for fracture functions obtained from the factorization of collinear singularities in the $\mathcal{O}(\alpha_s^2)$ SIDIS cross section.

In the case where a gluon is the fragmenting particle, we obtained

$$
\begin{aligned}
&P^{(1)}_{gq\leftarrow q}(u,v)\\
&\quad = C_F{}^2 \Bigg\{ -\frac{3-u}{1+v} + \frac{2\,u^2}{(1-u\,v)^2} + 2\,u + \frac{2}{(1+v)^2} - \frac{(3-u)\,u}{1-u\,v} + \Bigg[\frac{u^2}{(1-u\,v)^2} - (1+v)^{-2} - \frac{1+u}{1+v}\\
&\qquad + \frac{u\,(1+u)}{1-u\,v}\Bigg] \left(\log(u) + \log(a_f - v) - \log(v)\right) + \Bigg[2\,p_{gq\leftarrow q}(u) \left(\frac{1}{v} + \frac{u}{1-u-u\,v}\right) - 2\,u\\
&\qquad - \frac{2}{(1+v)^2} - \frac{2\,(1+u)}{1+v}\Bigg] \left(\log(1+v) + \log(1-u\,v)\right) + \Big[u - 2\,(1-u)\,\log(u) + 2\,p_{gq\leftarrow q}(u)\\
&\qquad \times \big(\zeta(2) - Li_2(u) - 2\,\log(1-u)\,\log(u)\big)\Big]\,\delta(a_f - v)\Bigg\}\\
&\quad + C_A\,C_F \Bigg\{ \frac{2\,u\,\left(2 - 3\,u + 2\,u^2\right)}{(1-u)^2} - \frac{2\,(1-u)}{v} - \frac{4\,u^2\,\left(1+u+u^2\right)\,v}{(1-u)^3} + \frac{4\,u^3\,\left(1+u+u^2\right)\,v^2}{(1-u)^4}\\
&\qquad + \left[\frac{u\,\left(11 - 2\,u + 7\,u^2\right)}{(1-u)^2} + p_{gq\leftarrow q}(u)\,\left(\frac{4\,u^3\,v^2}{(1-u)^3} - \frac{7}{v} - \frac{4\,u^2\,v}{(1-u)^2} - \frac{3\,u}{1-u-u\,v}\right)\right] \log(1-u)\\
&\qquad + \left[-\frac{u\,\left(9 - 4\,u + 3\,u^2\right)}{(1-u)^2} + \frac{u^2\,\left(3 - 2\,u + 3\,u^2\right)\,v}{(1-u)^3} + p_{gq\leftarrow q}(u)\,\left(\frac{7}{v} - \frac{2\,u^3\,v^2}{(1-u)^3} + \frac{2\,u}{1-u-u\,v}\right)\right]\\
&\qquad \times \log(u) + \left[-\frac{u\,\left(5 + 3\,u^2\right)}{(1-u)^2} + \frac{u^2\,(1+u)^2\,v}{(1-u)^3} + p_{gq\leftarrow q}(u)\,\left(\frac{3}{v} - \frac{2\,u^3\,v^2}{(1-u)^3}\right)\right] \log(a_f - v)\\
&\qquad + \left[-\frac{2\,u\,\left(3 - u + 2\,u^2\right)}{(1-u)^2} + \frac{u^2\,\left(3 - 2\,u + 3\,u^2\right)\,v}{(1-u)^3} + p_{gq\leftarrow q}(u)\left(\frac{4}{v} - \frac{2\,u^3\,v^2}{(1-u)^3} + \frac{3\,u}{1-u-u\,v}\right)\right]\\
&\qquad \times \log(v) + \left[\frac{u\,(1+u+u\,v)}{1-u} - p_{gq\leftarrow q}(u)\frac{u}{1-u-u\,v}\right] \log(1+v) + \left[\frac{u\,(2-u\,v)}{1-u} - \frac{p_{gq\leftarrow q}(u)}{v}\right]\\
&\qquad \times \log(1-u\,v) - \Big[2\,(1-u) + 4\,p_{gq\leftarrow q}(u)\,\log(a_f)\Big] \left(\frac{1}{a_f - v}\right)_{+v[0,\underline{a_f}]}\\
&\qquad + 4\,p_{gq\leftarrow q}(u)\left(\frac{\log(a_f - v)}{a_f - v}\right)_{+v[0,\underline{a_f}]} + \Big[2\,p_{gq\leftarrow q}(u)\,\left(\mathrm{Li}_2(u) + \log(1-u)\,\log(u)\right)\\
&\qquad - u\Big]\,\delta(a_f - v)\Bigg\},
\end{aligned}
\tag{63}
$$

where

$$
p_{qg\leftarrow q}(x) = \frac{1+x^2}{1-x}\,,\ \text{and}\ \ a_f = \frac{1-u}{u}\,. \tag{64}
$$

For the kernels relating different quark species, comig from the quark fragmenting processes, we obtained

$$
\begin{aligned}
P^{(1)}_{q_j q_i\leftarrow q_i}(u,v) = C_F\,T_F \Bigg\{&2\,u^2\,\frac{\left(1+u+u^2\right)\,v - u^3\,\left(1-v+v^2\right) - \left(1-u+u^2\right)\,\left(1+4\,u\,v+2\,u\,v^2\right) + 2\,p_{q\leftarrow g}(u)}{(1-u)^4}\\
&+ p_{gq\leftarrow q}(u)\,\left(\log(a_f - v) + \log(v) - \log(a_f) - \log(1-u)\right)\\
&\times \frac{u\,\left(1 - 2\,u\,(1+v) + u^2\,p_{q\leftarrow g}(v)\right)}{(1-u)^3}\Bigg\},
\end{aligned}
\tag{65}
$$

$$
P^{(1)}_{q_j \bar{q}_j\leftarrow q_i}(u,v) = C_F\,T_F \Bigg\{-2\,\frac{u^2\,v\,(1+v)^2 + (1-v)^2\,\left(1 - u\,(1+v)\right)}{u\,(1+v)^4} + \big(\log(u) + \log(a_f - v) - \log(v)
$$

$$+ 2\,\log(1+v)\big)\frac{\left(1+v^2\right)\,\left(u^2\,(1+v)^2+2\,\left(1-u\,(1+v)\right)\right)}{u\,(1+v)^4}\Bigg\}\,, \tag{66}$$

$$\begin{aligned}P^{(1)}_{q_i q_j\leftarrow q_i}(u,v) = C_F\,T_F\,\Bigg\{&2\,u^2\,\frac{u\,\left(2+u^2\right)\,v^2+7\,u\,v\,(1-u-u\,v)-(1-u)\,\left(1+v+u\,v+u^2\,v^3\right)}{(1-u\,v)^4}\\ &+\left[\log(u)+\log(a_f-v)+\log(v)-2\,\log\left(\frac{1-u-u\,v}{1-u\,v}\right)\right]\frac{u\,\left(1+u^2\,v^2\right)}{(1-u\,v)^4}\\ &\times\left(1+u^2\,v^2-2\,(1-u)\,u\,(1+v)\right)\Bigg\}\,,\end{aligned} \tag{67}$$

where

$$p_{q\leftarrow g}(x) = 1-2x+2x^2\,. \tag{68}$$

Finally, for the kernels relating quarks and antiquarks of the same flavor,

$$\begin{aligned}P^{(1)}_{\bar{q}q\leftarrow q}(u,v) = C_F\,T_F\,\Bigg\{&-\frac{2\,u\,\left(1+2\,u^2\right)}{3\,(1-u)^2}+\frac{4\,u^2\,\left(4+3\,u+5\,u^2\right)\,v}{3\,(1-u)^3}-\frac{2\,u^3\,\left(3+2\,u+3\,u^2\right)\,v^2}{(1-u)^4}-\frac{8}{u\,(1+v)^4}\\ &+\frac{8\,(1+u)}{u\,(1+v)^3}-\frac{2\,\left(3+10\,u-3\,u^2\right)}{3\,u\,(1+v)^2}+\frac{4\,(1-2\,u)}{3\,(1+v)}+\Bigg[-\frac{2\,u\,\left(5-3\,u+4\,u^2\right)}{3\,(1-u)^2}\\ &+\frac{2\,u^2\,\left(7-2\,u+7\,u^2\right)\,v}{3\,(1-u)^3}-\frac{4\,u^3\,\left(1+u^2\right)\,v^2}{(1-u)^4}+\frac{2\,\left(1+u^2\right)}{3\,(1-u)\,(1+v)}\Bigg]\log(1-u)\\ &+\Bigg[\frac{2\,u\,\left(5-6\,u+4\,u^2\right)}{3\,(1-u)^2}-\frac{4\,u^2\,\left(2-u+2\,u^2\right)\,v}{3\,(1-u)^3}+\frac{2\,u^3\,\left(1+u^2\right)\,v^2}{(1-u)^4}+\frac{4}{u\,(1+v)^4}\\ &-\frac{4\,(1+u)}{u\,(1+v)^3}+\frac{2\,(1+u)^2}{u\,(1+v)^2}-\frac{4\,\left(2-u^2\right)}{3\,(1-u)\,(1+v)}\Bigg]\log(u)+\Bigg[\frac{2\,u\,\left(5-6\,u+4\,u^2\right)}{3\,(1-u)^2}\\ &-\frac{4\,u^2\,\left(2-u+2\,u^2\right)\,v}{3\,(1-u)^3}+\frac{2\,u^3\,\left(1+u^2\right)\,v^2}{(1-u)^4}+\frac{4}{u\,(1+v)^4}-\frac{4\,(1+u)}{u\,(1+v)^3}+\frac{2\,(1+u)^2}{u\,(1+v)^2}\\ &-\frac{4\,\left(2-u^2\right)}{3\,(1-u)\,(1+v)}\Bigg]\log(a_f-v)+\Bigg[\frac{2\,u^2}{(1-u)^2}-\frac{2\,u^2\,\left(1+u^2\right)\,v}{(1-u)^3}+\frac{2\,u^3\,\left(1+u^2\right)\,v^2}{(1-u)^4}\\ &-\frac{4}{u\,(1+v)^4}+\frac{4\,(1+u)}{u\,(1+v)^3}-\frac{2\,(1+u)^2}{u\,(1+v)^2}+\frac{2\,(1+u)}{1+v}\Bigg]\log(v)+\Bigg[\frac{2\,(1-2\,u)\,u}{3\,(1-u)}+\frac{2\,u^2\,v}{3\,(1-u)}\\ &+\frac{8}{u\,(1+v)^4}-\frac{8\,(1+u)}{u\,(1+v)^3}+\frac{4\,(1+u)^2}{u\,(1+v)^2}-\frac{2\,\left(5-7\,u^2\right)}{3\,(1-u)\,(1+v)}\Bigg]\log(1+v)\Bigg\}\,,\end{aligned} \tag{69}$$

and

$$\begin{aligned}&P^{(1)}_{qq\leftarrow q}(u,v)+P^{(1)}_{q\bar{q}\leftarrow q}(u,v)\\ &\quad= C_F\,T_F\,\Bigg\{-\frac{2\,u\,\left(2+u^2\right)}{3\,(1-u)^2}+\frac{4\,u^2\,\left(5+3\,u+4\,u^2\right)\,v}{3\,(1-u)^3}-\frac{2\,u^3\,\left(3+2\,u+3\,u^2\right)\,v^2}{(1-u)^4}-\frac{8}{u\,(1+v)^4}\\ &\qquad+\frac{8\,(1+u)}{u\,(1+v)^3}-\frac{2\,\left(3+14\,u-3\,u^2\right)}{3\,u\,(1+v)^2}-\frac{4\,(-2+u)}{3\,(1+v)}-\frac{16\,u^3}{(1-u\,v)^4}+\frac{16\,u^2\,(1+u)}{(1-u\,v)^3}\\ &\qquad-\frac{4\,u\,\left(1+4\,u-u^2\right)}{(1-u\,v)^2}+\frac{2\,u\,\left(5-12\,u+6\,u^2\right)}{3\,(1-u)\,(1-u\,v)}+\Bigg[-\frac{2\,u\,\left(4-u+3\,u^2\right)}{3\,(1-u)^2}\\ &\qquad+\frac{2\,u^2\,\left(5+2\,u+5\,u^2\right)\,v}{3\,(1-u)^3}-\frac{4\,u^3\,\left(1+u^2\right)\,v^2}{(1-u)^4}+\frac{2\,u\,\left(1+u^2\right)}{3\,(1-u)\,(1-u\,v)}\Bigg]\log(1-u)\end{aligned}$$

$$
\begin{aligned}
&+\left[\frac{2\,u\,(2+u)}{3\,(1-u)^2}-\frac{2\,u^2\,\left(1+4\,u+u^2\right)\,v}{3\,(1-u)^3}+\frac{2\,u^3\,\left(1+u^2\right)\,v^2}{(1-u)^4}+\frac{4}{u\,(1+v)^4}-\frac{4\,(1+u)}{u\,(1+v)^3}\right.\\
&+\frac{2\,(1+u)^2}{u\,(1+v)^2}-\frac{2\,(1+u)}{1+v}-\frac{8\,u^3}{(1-u\,v)^4}+\frac{8\,u^2\,(1+u)}{(1-u\,v)^3}-\frac{4\,u\,(1+u)^2}{(1-u\,v)^2}\\
&\left.+\frac{2\,u\,\left(5-8\,u^2\right)}{3\,(1-u)\,(1-u\,v)}\right]\log(u)+\left[\frac{2\,u^2}{(1-u)^2}-\frac{2\,u^2\,\left(1+u^2\right)\,v}{(1-u)^3}+\frac{2\,u^3\,\left(1+u^2\right)\,v^2}{(1-u)^4}\right.\\
&+\frac{4}{u\,(1+v)^4}-\frac{4\,(1+u)}{u\,(1+v)^3}+\frac{2\,(1+u)^2}{u\,(1+v)^2}-\frac{2\,(1+u)}{1+v}-\frac{8\,u^3}{(1-u\,v)^4}+\frac{8\,u^2\,(1+u)}{(1-u\,v)^3}\\
&\left.-\frac{4\,u\,(1+u)^2}{(1-u\,v)^2}+\frac{2\,u\,\left(5-6\,u^2\right)}{3\,(1-u)\,(1-u\,v)}\right]\log(a_f-v)+\left[\frac{2\,u\,\left(4-4\,u+3\,u^2\right)}{3\,(1-u)^2}\right.\\
&-\frac{4\,u^2\,\left(1+u+u^2\right)\,v}{3\,(1-u)^3}+\frac{2\,u^3\,\left(1+u^2\right)\,v^2}{(1-u)^4}-\frac{4}{u\,(1+v)^4}+\frac{4\,(1+u)}{u\,(1+v)^3}-\frac{2\,(1+u)^2}{u\,(1+v)^2}\\
&\left.+\frac{2\,(1+u)}{1+v}+\frac{8\,u^3}{(1-u\,v)^4}-\frac{8\,u^2\,(1+u)}{(1-u\,v)^3}+\frac{4\,u\,(1+u)^2}{(1-u\,v)^2}-\frac{2\,u\,\left(8-5\,u^2\right)}{3\,(1-u)\,(1-u\,v)}\right]\log(v)\\
&+\left[2\,u+\frac{8}{u\,(1+v)^4}-\frac{8\,(1+u)}{u\,(1+v)^3}+\frac{4\,(1+u)^2}{u\,(1+v)^2}-\frac{2\,\left(7+12\,u+7\,u^2\right)}{3\,(1+u)\,(1+v)}\right.\\
&\left.-\frac{2\,u\,\left(1+u^2\right)}{3\,(1+u)\,(1-u\,v)}\right]\log(1+v)+\left[\frac{2\,(5-6\,u)\,u}{3\,(1-u)}-\frac{2\,u^2\,v}{3\,(1-u)}-\frac{2\,\left(1+u^2\right)}{3\,(1+u)\,(1+v)}\right.\\
&+\frac{16\,u^3}{(1-u\,v)^4}-\frac{16\,u^2\,(1+u)}{(1-u\,v)^3}+\frac{8\,u\,(1+u)^2}{(1-u\,v)^2}+\frac{4\,u^2}{3\,(1+u)\,(1-u\,v)}\\
&\left.\left.-\frac{28\,u\,\left(1+2\,u+u^2\right)}{3\,(1+u)\,(1-u\,v)}\right]\log(1-u\,v)\right\}.
\end{aligned}
\tag{70}
$$

As mentioned in Section III, in this last case, only the combination $P^{(1)}_{qq\leftarrow q}(u,v)+P^{(1)}_{q\bar{q}\leftarrow q}(u,v)$ can be extracted. The remaining quark initiated kernels can be obtained from the charge conjugation relations given in that Section.

---

[1] L. Trentadue and G. Veneziano, *Phys. Lett.* B323, (1993) 201.
[2] D. Graudenz, *Nuc. Phys.* B432 351 (1994)
[3] M. Grazzini, L. Trentadue, G. Veneziano, *Nucl. Phys.* B519, (1998) 394;
M. Grazzini, *Phys. Rev.* D57, (1998) 4352.
[4] D. de Florian, C. A. García Canal and R. Sassot, *Nuc. Phys.* B470, (1996) 195.
[5] see C.A. Garcia Canal, R. Sassot, *Int. Jour. Mod. Phys.* A15 (2000) 3587.
[6] D. de Florian and R. Sassot, *Phys. Rev.* D58, (1998) 054003.
[7] D. de Florian, O. A. Sampayo, R. Sassot, *Phys. Rev.* D57 (1998) 5803.
[8] D. de Florian, R. Sassot, *Phys. Rev.* D62 (2000) 094025.
[9] D. de Florian and R. Sassot, *Phys. Rev.* D56, (1997) 426.
[10] A. Daleo, C. García Canal, R. Sassot, *Nucl. Phys.* B662, 334 (2003).
[11] E.B. Zijlstra, W.L. van Neerven, *Nuc. Phys.* B383 (1992) 525.
[12] C. G. Bollini, J. J. Giambiagi, *Nuovo Cimento* 12B, (1972) 20, G. 't Hooft, M. Veltman, *Nucl. Phys.* B44, (1972) 189.
[13] S. Wolfram, *Mathematica* Third Edition, Cambridge University Press 1996.
[14] M. Jamin, M.E. Lautenbacher, *Comp. Phys. Comm.* 74 (1993) 265.
[15] W. Beenakker, H. Kuijf, W. L. van Neerven and J. Smith, *Phys. Rev.* D40 (1989) 54. W. Beenakker, Ph. D. Thesis, Leiden, 1989.
[16] G. Passarino, M. Veltamn, *Nucl. Phys.* B160 (1979) 151.
[17] G. Kramer, B. Lampe, *Z. Phys.* C34 (1987) 497.
[18] T. Matsuura, W. L. van Neerven, *Z. Phys* C38 (1988) 623.
[19] T. Matsuura, S. C. van der Marck, W. L. van Neerven, *Nuc. Phys.* B319 (1989) 570.
[20] V. N. Gribov, L. N. Lipatov, *Sov. J. Nucl. Phys.* 15 (1972) 438, G. Altarelli, G. Parisi, *Nucl. Phys.* B126 (1977) 298.
[21] H. Holtman, et al., Phys.Lett.B338 (1994) 363.

[22] M. Glück, E. Reya, A. Vogt, *Eur. Phys. J.* C5 (1998) 461.
[23] S. Kretzer, *Phys. Rev.* D62 (2000) 054001.
[24] See for example, *NLO QCD Corrections to Polarized Photo and Hadroproduction of Heavy Quarks*, I. Bojak Ph.D Thesis, Universitet Dortmund (2000).
[25] W.L. van Neerven, *Nuc.Phys.* B268 (1986) 453.
[26] Higher transcendental functions, vol. 1, The Bateman manuscript. Ed. A. Erdèly. McGraw-Hill, New York, 1953.